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The work done and the number of subjects studied goes beyond what was promised in the original two-year proposal. The proposal focused on single-sensor parameter estimation for ARMA signals in noise. We solved several open problems in this area and added sine wave noise. We extended the work to sensor array estimation algorithms for localization (tracking applications). The music estimator was developed and compared to the maximum-likelihood estimator. Closed form expressions for the Cramer-Rao bound weve discovered for certain cases of significance also is the thesis of David Starer written with AFOSR support, that provides up-dating of the roots of an $n^{th}$ order polynomial in $0(u^2)$ time, and numerous applications.

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# Final Report
# Research Under Grant AFOSR-88-0080 November 1, 1987 to January 31, 1990

## 1 Introduction

This report describes the work supported by the Air Force Office of Scientific Research under Grant AFOSR-88-0080 from November 1, 1987 to January 31, 1990. The grant supported the work of the Principal Investigator Professor Arye Nehorai and of his student David Starer at Yale University.

The amount of work we have done and the number of subjects studied in one year was far beyond what was promised in the original two year proposal [1]. The original proposal focused on the development and analysis of single-sensor parameter estimation schemes for ARMA signals in noise. In the single sensor problem, we solved several new problems for these signals and also added sine wave signals in noise. Furthermore, we extended the research to multisensor (or sensor array) estimation algorithms which are useful for localization of energy radiating sources. Examples are localization of sound sources such as helicopters and electromagnetic sources such as radio transmissions. All of the proposed algorithms have been tested by computer simulation to verify their operation.

A testimony to the significance and recognition of our recent research is that we were invited to publish the article [11] as a chapter in a forthcoming book edited by S. Haykin [15]. The article [11] recently received the IEEE Signal Processing Society's Senior Award.

The research results obtained this year are described in detail in the references [2]–[14]. The work in all of these articles was performed with the support (either whole or partial) of the AFOSR grant, and this financial support is acknowledged in the articles themselves. The work is summarized in the following.

## 2 Single Sensor Algorithms

### 2.1 Algorithm Development

#### 2.1.1 Adaptive Algorithms for Constrained ARMA Signals in the Presence of Noise [2]

A convenient model for time-series analysis and system identification is the autoregressive moving-average (ARMA) or rational transfer function model: In time-series analysis, a measured signal can be modeled as the output of a rational transfer function driven by white noise. The estimated model can then be used to determine the signal spectrum. In system identification, an unknown system can be modeled as a rational transfer function whose parameters can be estimated from measurements of the input and output signals using ARMA identification algorithms. Knowledge of these parameters can be used, for example, by an adaptive regulator to control the system.

Most existing ARMA estimators employ "unconstrained" rational models. However, in many applications the measured signal is known to have special properties, and it is preferable to constrain the model to conform to these properties. Furthermore, most existing ARMA estimators ignore the contamination of the signal by noise, and this leads to incorrect estimates. To solve these problems, we developed in [2] a general family of ARMA adaptive algorithms which utilize *a priori* known information concerning the signal's properties and solve the problems associated with nonlinearities which arise from the presence of noise. These algorithms can track time-varying parameters, are more accurate and are computationally more efficient than the unconstrained ARMA algorithms. Applications of these algorithms include signal analysis in multipath scenarios, image deblurring, structured signal (e.g. band-pass) spectrum analysis.

### 2.1.2 Adaptive Pole Estimation [3]

In many ARMA spectral estimation problems, the parameters of interest are the poles (or singularities) of the rational transfer function. These are useful, for example, for spectral peak estimation, robust speech communication, and estimation of direction of arrivals (DOAs) of energy waves impinging on a sensor array. Thus there exists a need for an algorithm which can satisfy the simultaneous requirements of a) directly providing estimates of a system's poles and of b) providing new pole estimates as each data sample is received. Our research in [3] has provided an algorithm which satisfies the above requirements. In this method, the system is parametrized directly in terms of its poles. Convergence analysis is provided and it is shown that the algorithm converges to the true parameters. The research has already provided some spinoff results in the form of new polynomial factorization algorithms described below (see also [4]).

### 2.1.3 Polynomial Factorization Algorithms for Adaptive Root Estimation [4]

Polynomial factorization is often required by signal processing and system identification algorithms. Examples include speech processing, frequency estimation of multiple complex exponential signals in noise, and source localization using a uniform linear array of sensors. Conventional polynomial factorization algorithms cannot be used to track the roots associated with time-varying coefficients, i.e., if the coefficients change slightly, the entire factorization procedure must be restarted.

Our research in [4] has produced a new efficient algorithm which overcomes the above difficulties. It estimates all roots simultaneously so that deflation is not needed. This eliminates the necessity for estimating individual roots to maximum accuracy at each iteration. The method has guaranteed global convergence and exhibits a quadratic convergence rate close to the true root vector.

### 2.1.4 Non-Iterative Optimal Min-Max Instrumental Variable Method for System Identification [5]

One of the common problems in system identification is that the noise characteristics may not be known or may vary from application to application. In [5] we have developed a new system identification algorithm that is robust against unknown noise properties. The method is an optimal min-max instrumental variable (IV) method that gives the smallest estimation error variance in the worst noise case from a prespecified class of noises. Implementation of the new method does not require knowledge of system or noise parameters and therefore can be done *without iteration.* It is useful for example for deadbeat controllers where there is no need for the noise parameters to be estimated. This method was applied successfully to both artificial and real data from gas furnace measurements.

## 2.2 Performance Analysis

Performance analysis includes derivation of the error covariance matrices of estimation algorithms and derivation of generic bounds on estimation accuracy. A common bound is the Cramér-Rao bound (CRB) which is a lower bound on the estimation error covariance matrix of any unbiased estimator. The CRB is useful for evaluating the quality of algorithms and for gaining insight into the problem of interest.

### 2.2.1 Statistical Analysis of Two Non-Linear Least-Squares Estimators of Sine Wave Parameters in the Colored Noise Case [6]

The main difficulty in deriving the covariance matrix of estimation algorithms stems from their estimation nonlinearities. In [6] we have provided a statistical analysis of two nonlinear least-squares estimators (NLSEs) of sine wave parameters in the colored noise case. These estimators are the basic NLSE, which ignores the possible correlation of the noise, and the optimal NLSE, which estimates the noise correlation (appropriately parametrized) as well as the sine-wave parameters. It is shown that these two NLS estimators have the *same accuracy* in large samples. This result provides complete justification for preferring the computationally less expensive basic NLSE over the optimal NLSE. Both estimators are shown to achieve the Cramér-Rao bound as the sample size increases. A simple explicit expression for the asymptotic CRB matrix is provided, which should be useful in studying the performance of sine-wave parameter estimators designed to work in the colored noise case.

### 2.2.2 Performance Analysis of the Pisarenko Harmonic Decomposition Method [7]

One of the well known methods for sine wave frequency estimation is the Pisarenko method (see corresponding reference in [7]). The method consists of determining the minimum eigenvalue of the data covariance matrix and its associated eigenfilter zeros. It has been very popular due to its computational simplicity. In [7] we provide a self-contained statistical analysis of this method. An explicit formula is provided for the asymptotic covariance matrix of the method. This covariance is then compared with the Cramér-Rao bound and the Yule-Walker method (see [7]). Our results show that both the Pisarenko and the Yule-Walker methods are quite inefficient in the statistical sense. Their error variances are much larger than the CRB. It is also shown that the variances of these two methods increase faster than the CRB when the signal-to-noise ratio decreases. Since both the Pisarenko and the Yule-Walker methods are quite inefficient statistically, they are attractive only when computational simplicity is a must.

### 2.2.3 On the Stability of Least-Squares Models Fitted to Multivariable Input-Output Data [8]

The stability of least-squares models is important in many aspects of parameter estimation and has been the subject of intensive study leading to classical results for time series and other special cases. However, in the case of multivariable dynamic systems with a control (or exogenous) input, no such results exist except for certain very special cases.

We have studied *general multivariable* systems and models of *arbitrary (finite) order* in [8]. The work has resulted in a theorem concerning the stability of such systems subject to a set of easily satisfied assumptions which are generally assured to hold in practice. We have also developed some easily-computed upper bounds on the moduli of the poles of the systems.

# 3 Sensor Array Processing

Sensor array processing is a common name for several problems associated with arrays of sensors. The sensors may be electromagnetic antennas, microphones, geophones, underwater sonars, etc. These sensors are arranged in an arbitrary geometry to form a *sensor array*. The sensor array records signal waveforms of distant emitters. These in turn are used to estimate unknown emitter parameters, such as directions of arrivals, location, signal waveforms, etc. The great interest in sensor array processing stems from their wide applicability in radar, sonar, radio and microwave communication, seismology and hydroacoustics. An interesting note is that damped/undamped sine wave parameter estimation in noise turns out to be a special case of the sensor array processing problem.

The sensor output data is modeled as a superposition of individual wave forms. In the narrow-band case (i.e. when the power of all the emitter signals is in the same narrow frequency band) the sensor data model is:

$$y(t) = A(\theta)x(t) + e(t) \qquad t = 1, 2, \ldots, N \tag{1}$$

where $\{y(t)\}$ are the observed sensor data vectors, $\{x(t)\}$ are the unknown signal wave form vectors, and $e(t)$ is an additive noise. The matrix $A(\theta)$ models the characteristics of the array (e.g. geometry and sensor directivity). Based on the sensor observations $\{y(t)\}$, the parameters of interest ($\theta$ and $\{x(t)\}$) are estimated.

Two classes of methods for estimating the unknown parameters of (1) have received significant attention in recent years. The first is the multiple signal characterization (MUSIC) method which is based on the eigendecomposition of the data sample covariance matrix and is in fact an ad-hoc method. The second is the maximum likelihood (ML) which is based on statistical foundations.

In the following we summarize our research results in the area of sensor array processing on these algorithms and the corresponding CRB.

## 3.1 Algorithm Development

### 3.1.1 Maximum Likelihood Estimation of Exponential Signals in Noise Using a Newton Algorithm [9]

The problem of estimating the direction of arrivals of far-field source wave forms impinging on uniform linear arrays and the related one of estimating the parameters of multiple exponential signals in noise is important in many applications of signal processing. As yet, no optimal estimation methods tailored to this problem with guaranteed convergence have been found since the estimation involves the difficult problem of maximizing the highly nonlinear likelihood function. Our research in [9] presents an ML solution to the problem. Using the properties of shift matrices we derive simple, exact, closed-form expressions for the gradient vector and the Hessian matrix. Based on these results, a Newton algorithm is presented for the estimation of source DOAs and multiple exponential signals in noise. The main advantages of this algorithm include ease of implementation with matrix software packages (such as MATLAB), as well as fast and guaranteed convergence to a local maximum of the likelihood function. A globally convergent ML algorithm is currently being developed.

### 3.1.2 Conditional and Unconditional Maximum Likelihood Estimation of the Parameters of Exponential Signals in Noise [10]

The Conditional Maximum Likelihood (CML) and Unconditional Maximum Likelihood (UML) cost functions (described more fully in Section 3.2.2) are highly nonlinear and multimodal with many false local minima. Thus estimation of source location parameters by minimization of these cost functions is a difficult problem. In the past, algorithms for UML and CML source localization were based on intuition rather than rigorously correct optimization techniques. These intuitive methods lack the very important property of guaranteed convergence. In our work in [10], we propose new algorithms with guaranteed local convergence for minimization of the CML and UML cost functions.

This work deals specifically with uniform linear arrays (ULAs). The structure of the uniform linear array allows the unconditional and conditional negative log likelihood functions to be reparametrized in terms of a polynomial model, and the coefficients of this polynomial become the parameters to be estimated. Projection onto the orthogonal complement of the space spanned by the columns of the steering matrix is required in the likelihood functions. With reparametrization, this projection is replaced by a projection onto the space spanned by the columns of a Toeplitz matrix with the desired polynomial coefficients along its diagonals. This fact is exploited to derive gradient-based source localization algorithms which are computationally efficient.

## 3.2 Performance Analysis

### 3.2.1 MUSIC, Maximum Likelihood and Cramér-Rao Bound [11]

Since the MUSIC is computationally simpler than the ML method, and since these methods are widely used, it has been of great interest and importance to evaluate the performance of these two methods, comparing them with one another and with the CRB.

Our paper [11] provides extensive studies of the performance of the MUSIC and ML methods, and analyzes their statistical efficiency. Closed-form expressions are derived for the MUSIC covariance matrix and the CRB for the estimation problem (1). It is shown that unless the emitter signals are uncorrelated (i.e. absence of multipaths) and the number of sensors is large, the MUSIC estimator cannot achieve the CRB. The relationship between the MUSIC and the ML is also investigated. It is shown that the MUSIC is a large sample (i.e. large $N$) realization of the ML estimate if and only if the emitter signals are uncorrelated. The paper also contains results on the resolvability of the MUSIC and ML algorithms for the problem of finding the DOAs of plane waves using a uniform linear array.

One of the well known results in estimation theory is that the ML method achieves the CRB asymptotically. Our results prove that, for the problem (1), the ML method cannot achieve the CRB unless the number of sensors is very large, even if the number of data ($N$) is large. This "shocking" result is explained in [11]. It raises the question: Do better estimators than the ML method exist for a finite number of sensors? This question has also been investigated.

### 3.2.2 MODE, Maximum Likelihood and Cramér-Rao Bound [12]

Parameter estimation based on maximum likelihood methods requires knowledge of (or assumptions concerning) the statistical distribution of the data to be analyzed. In sensor array processing, the data consists of superimposed signals corrupted by noise, and two statistical models of the signal generation mechanism are currently used. These are: 1) The conditional model (CM) in which the signals are assumed to be nonrandom, and 2) the unconditional model (UM) in which the signals are assumed to be random. Use of these two models leads to two different estimation algorithms (termed CML and UML respectively) and two different Cramér-Rao bounds ($B_c$ and $B_u$) on estimation accuracy. However, an algorithm derived under either one of the two model assumptions may be applied to data generated by the other model. A third estimation method, based on the asymptotic statistical properties of the sample covariance matrix eigendecomposition, is the method of direction estimation (MODE) and it too may be applied to data generated under the CM or UM.

One of the important issues investigated in our work reported in [12] has been the study of the statistical properties of estimates obtained from algorithms based on possibly incorrect signal generation models. In [12] we presented a rigorous derivation of the covariance matrices $C_{\mathrm{CML}}$, $C_{\mathrm{UML}}$ and $C_{\mathrm{MODE}}$ of the CML, UML and MODE estimates, as well as bounds $B_c$ and $B_u$ on the accuracy attainable under both signal models CM and UM. It was shown that CML, UML, MODE and many other DOA estimation methods yield estimates whose statistical properties are independent of the way in which the data is generated (i.e. nonrandom or random).

Study of the covariance matrices has shown $B_u$ is a lower bound on any consistent DOA estimates based on the sample covariance matrix. It was also shown that MODE is asymptotically equivalent to UML, and that both of these methods achieve $B_u$ asymptotically. The study also showed that CML is less efficient statistically than UML, and that $B_c$ is unattainable.

Both $B_c$ and $B_u$ decrease monotonically as the number of sensors or snapshots increases, and they increase as the number of sources increases. Also, as the number of sources or the signal-to-noise ratio increases, all matrices $C_{\mathrm{CML}}$, $C_{\mathrm{UML}}$, $C_{\mathrm{MODE}}$, $B_c$ and $B_u$ tend to the same limit matrix.

The results of this part indicate that the best methods to be used in highly correlated source scenarios (due to multipath effects) are MODE and UML. In these scenarios, the popular methods MUSIC and CML suffer severe loss in accuracy.

### 3.2.3 Consistency of DOA Estimation with Multipath and Few Snapshots [13]

A fundamentally important property required of estimation *algorithms* is consistency; i.e. that the estimate obtained should be equal to the true parameter vector for a large amount of data or under favorable conditions such as high signal-to-noise ratio. This topic of algorithmic consistency has not previously been studied in the context of DOA estimation.

In our work published in [13], we have derived conditions under which the CML and MODE algorithms lead to unique solutions. The results have shown that the conditions under which these two algorithms achieve unique, consistent estimates coincide with the conditions under which the problem itself has a unique solution. This, in turn, reduces to fundamental conditions on the array (specifically the number of sensors in relation to the number of sources). The work has also revealed that, of the two possible implementations of MODE, only one is guaranteed to give consistency.

### 3.2.4 Ph.D Thesis [14]

The author's student, David Starer, has recently completed his Ph.D thesis [14] with the financial support of the AFOSR. The thesis contains the details pertaining to several portions of this report. Its research led to the publications [3]–[4]), [9]–[10]), as well as a newly discovered algorithm which promises to yield some powerful results in the future. The new algorithm is based on homotopic theory, and provides a passive method of estimating both bearings and ranges of multiple sources. This is the first high-resolution method to be able to estimate these parameters with minimal computational complexity and guaranteed global convergence. Preliminary results are extremely encouraging, but a significant amount of study still needs to be devoted to this new subject.

# 4 Future Outlook

Our research on algorithm development and analysis supported by this grant is rapidly growing. Under this grant we have solved in [2]–[14] many important and difficult problems which existed in the signal processing and system identification areas. Most of these problems were not raised in the original proposal. However, there are many problems that we still have to address.

In the future we will continue the development and analysis of algorithms for signal processing and system identification. We will derive the ML covariance matrix for (1) and analyze methods other than MUSIC and the MLE, such as weighted MUSIC, ESPRIT and subspace rotation methods (SRMs). We will consider sensor array models that are more general than (1). For instance, models of near-field sources, correlated sensor noises, signals modeled through their covariances rather than their amplitudes, and finite data models. We will develop new algorithms and analyze their performance for these problems. A globally convergent algorithm for (1) will be developed. Optimal methods that could outperform the ML method will be developed for a finite number of sensors and data samples. Bounds that are more appropriate than the CRB will be considered for the problems of closely spaced emitters and damped (transient) signals.

The support of the Air Force Office of Scientific Research has been essential for the research in [2]-[14] and is gratefully acknowledged.

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